

# Learning When to Halt With Adaptive Computation Time

Alex Graves, Oriol Vinyals, Michael Figurnov, Rafal Jozefowicz

*Adaptive Computation Time With Recurrent Neural Networks*  
Graves, 2016

*Spatially Adaptive Computation Time for Residual Networks*  
Figurnov et. al, 2016

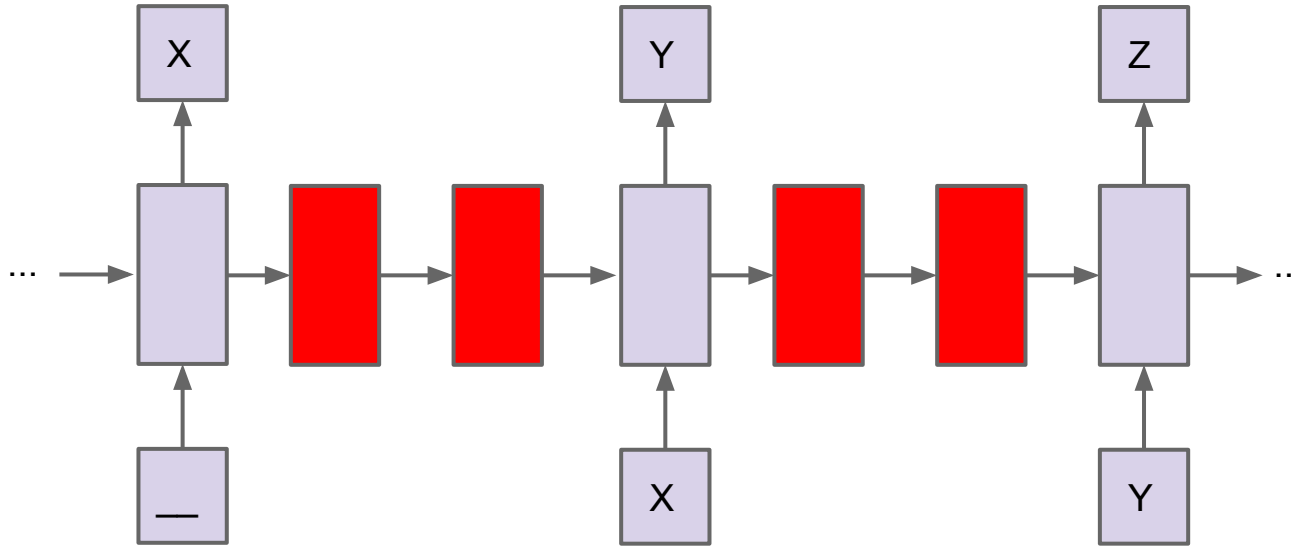
*Publish Me Soon!*

Vinyals, Graves, Raffel, Jozefowicz, 20??

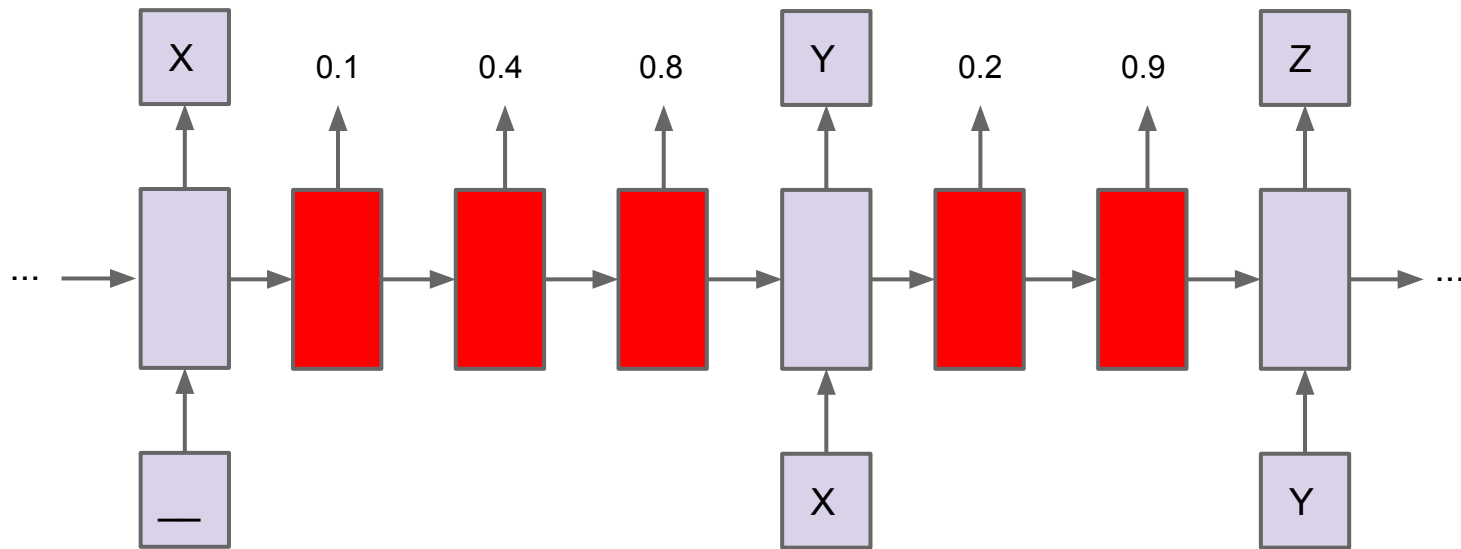
# Motivation

- At the moment the number of **steps of computation** an RNN gets for a given problem is determined by the data (sequence **length**) and the experimenter (network **depth**, **padding** in sequence...)
- Would prefer the net to **decide** how to long to **'ponder'** each input before it outputs an answer
- Clearly useful for **algorithmic** / **planning** type problems with a high variance in complexity (e.g. program induction, pathfinding...)
- Can also be more efficient for conventional tasks such as **machine translation**, **language modelling** and **image processing**
- More important for supervised learning than e.g. RL

# Fixed Computation Time



# Adaptive Computation Time (ACT)



# Adaptive Computation Time (ACT)

Add a *halting unit*  $h$  to the output

$$h_t^n = \sigma(W_h s_t^n + b_h)$$

Use this to define the *halt probability*  $p_t^n$  at ponder step  $n$

$$p_t^n = \begin{cases} R(t) & \text{if } n = N(t) \\ h_t^n & \text{otherwise} \end{cases}$$

Where  $N(t)$  is # updates at  $t$

$$N(t) = \min\{n' : \sum_{n=1}^{n'} h_t^n \geq 1 - \epsilon\}$$

And  $R(t)$  is the *remainder* at  $t$

$$R(t) = 1 - \sum_{n=1}^{N(t)-1} h_t^n$$

The final states and outputs at  $t$  are *weighted sums* (!)

$$s_t = \sum_{n=1}^{N(t)} p_t^n s_t^n \quad y_t = \sum_{n=1}^{N(t)} p_t^n y_t^n$$

# Limiting Computation Time

We always want answers as quick as possible, but can't tell in advance how long that will be (**halting problem**). ACT adds a **ponder cost**  $P(\mathbf{x})$  to the loss function and uses a **time penalty**  $\tau$  to trade off accuracy against speed

$$\hat{\mathcal{L}}(\mathbf{x}, \mathbf{y}) = \mathcal{L}(\mathbf{x}, \mathbf{y}) + \tau \mathcal{P}(\mathbf{x}) \qquad \mathcal{P}(\mathbf{x}) = \sum_{t=1}^T N(t) + R(t)$$

$P(\mathbf{x})$  is an upper bound on the **total computation**  $\sum_t N(t)$ . It is **discontinuous** when  $N(t)$  changes, but we just ignore that and minimise  $R(t)$ , which maximises the amount of halt probability mass assigned to steps  $< N(t)$ .

Minimising **expected** emission time **doesn't do this**

# Toy Experiments: Addition

1	3	6
0	9	8
3	2	4
8	-	5
-	-	0

Input seq.



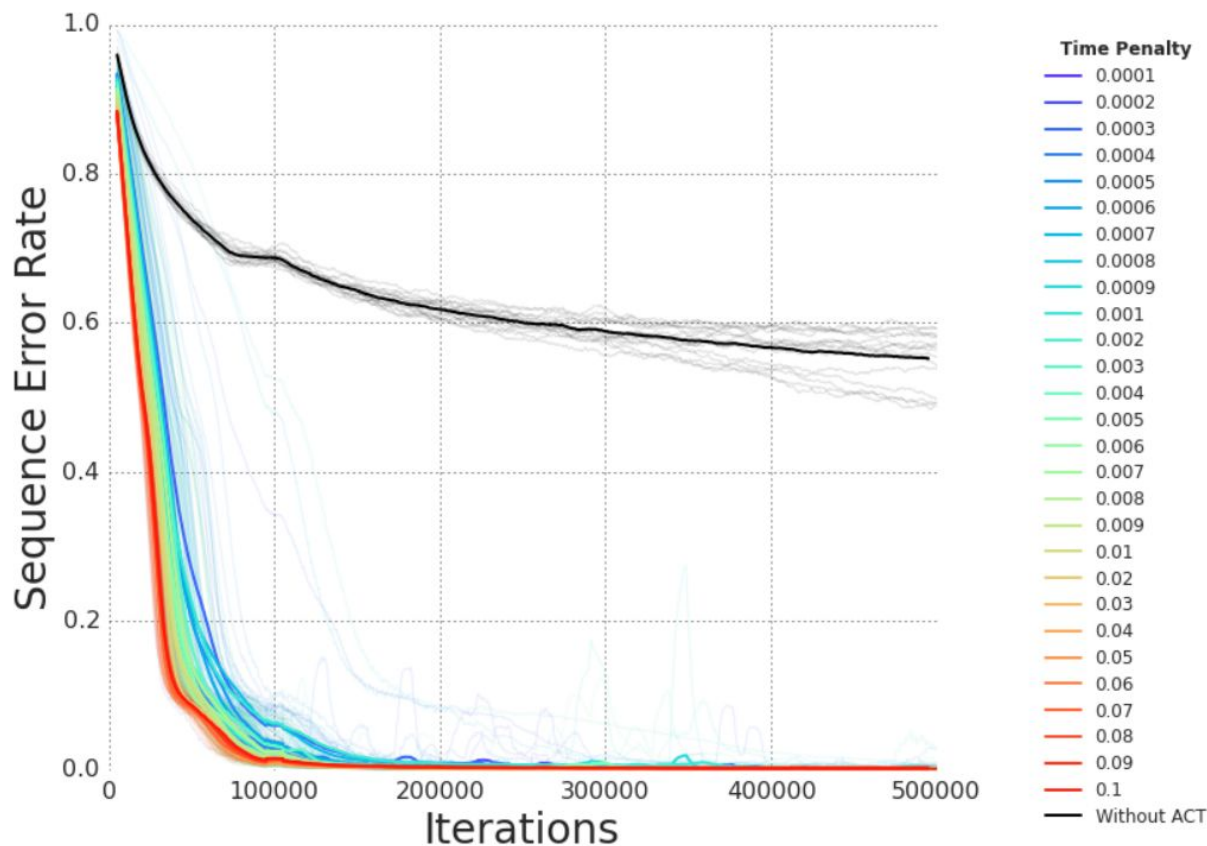
	1	6
	4	9
	3	8
	0	8
	*	0
	*	*

Target seq.

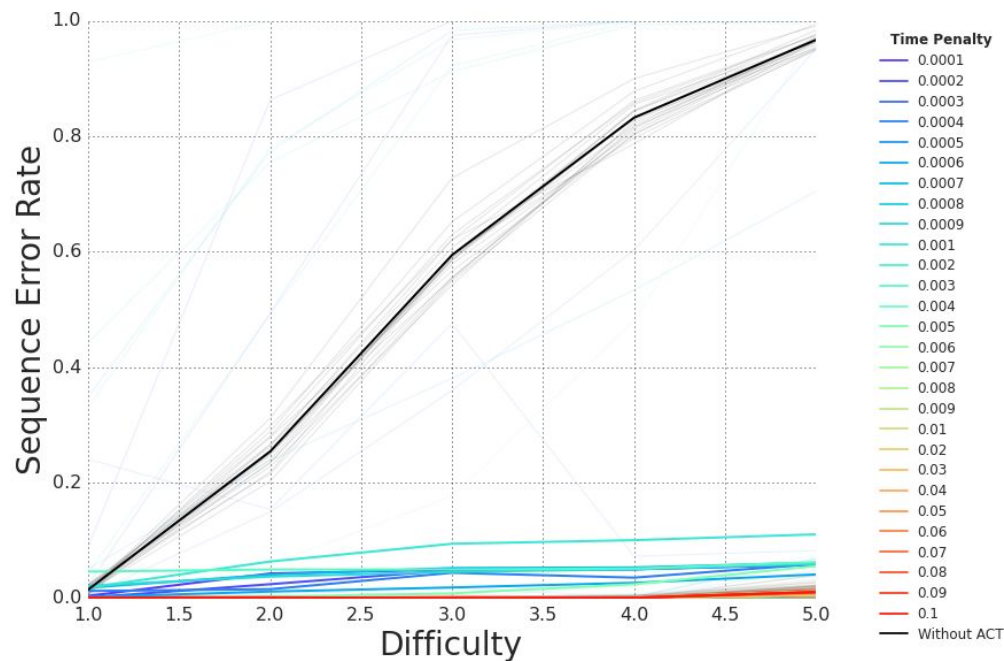
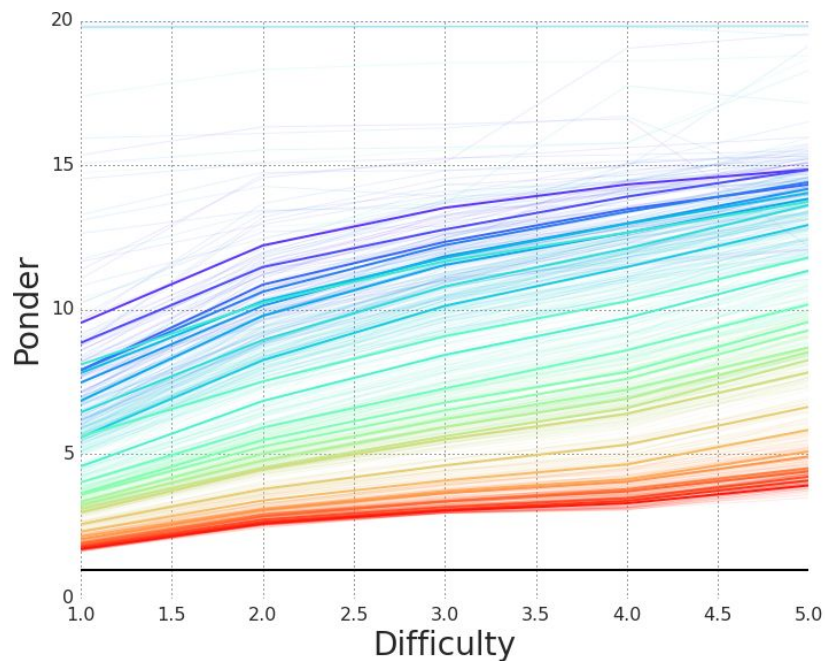


# Toy Experiments: Addition

Slide Credit:  
Alex Graves



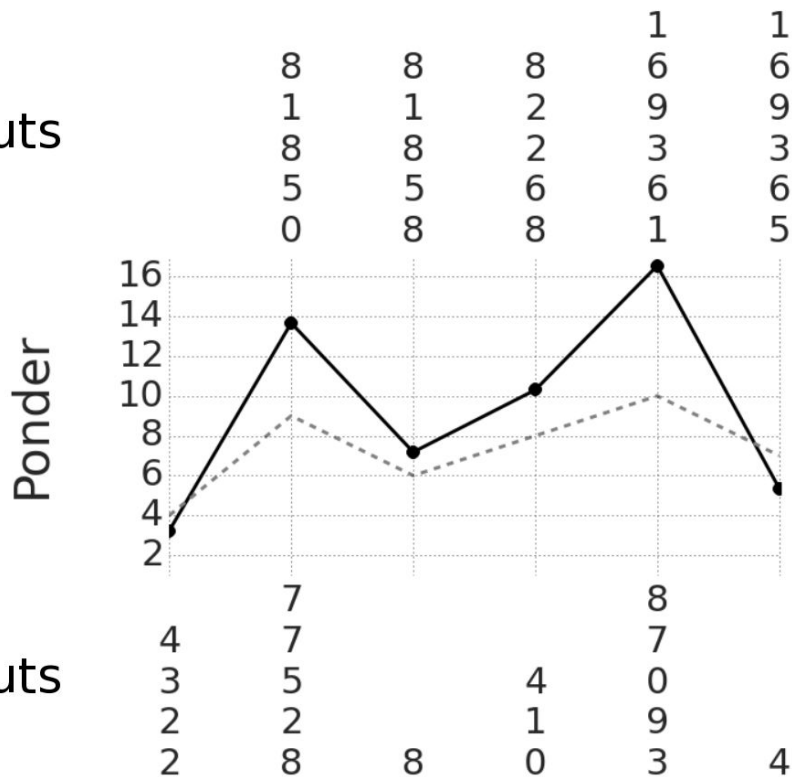
# Toy Experiments: Addition



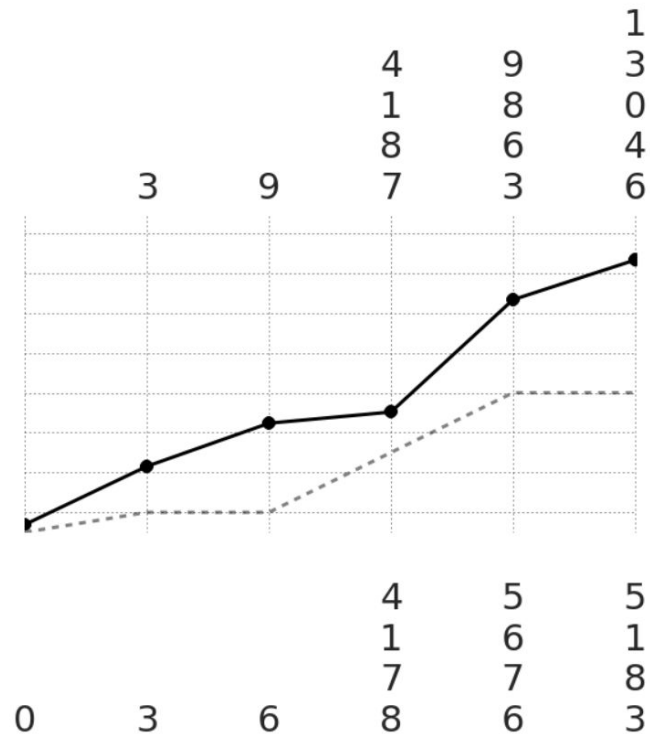
# Toy Experiments: Addition

Slide Credit:  
Alex Graves

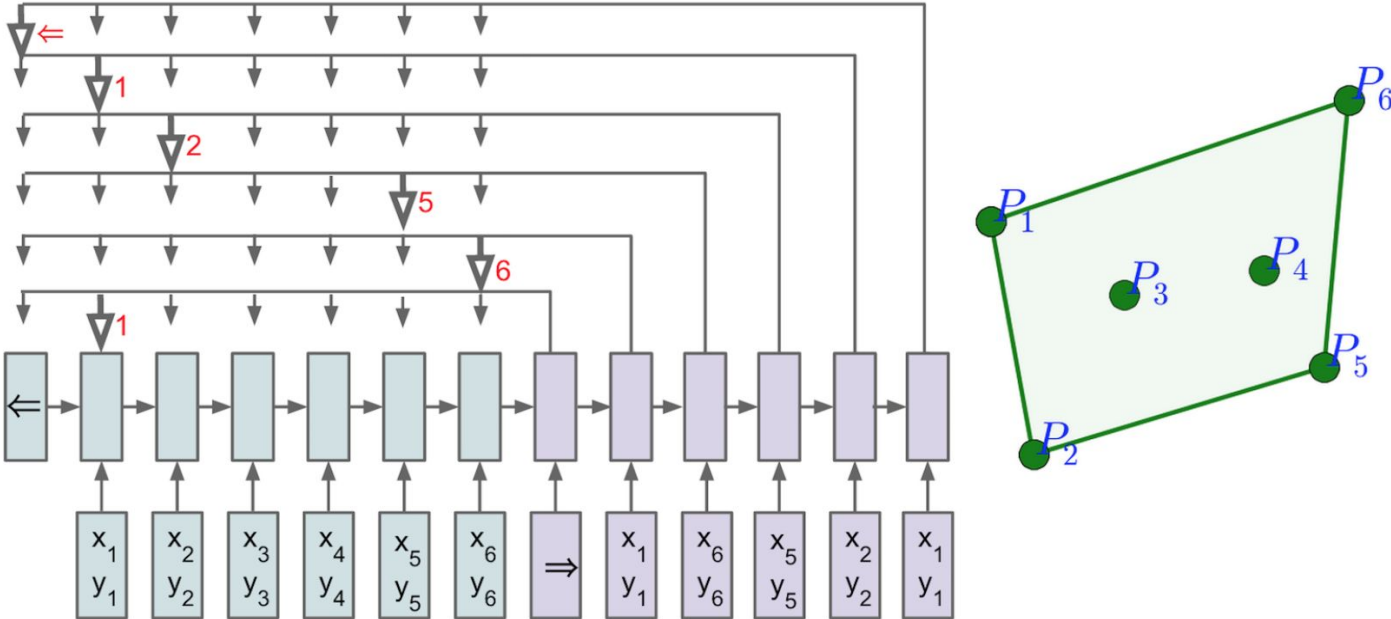
Outputs



Inputs



# Toy Experiments: PtrNets, TSP / ConvexHull



# Pointer Nets + ACT

Convex Hull (50): 73% accuracy -> **85%** accuracy

TSP (50):

Optimal: 5.7

Heuristic algorithm: 5.8

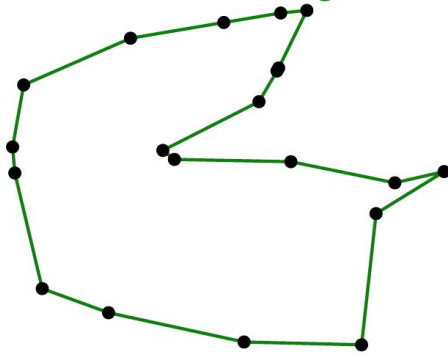
PTR-NETs (NIPS version): 6.1

PTR-NETs + ACT: **5.9**

(new ICLR17 submission)

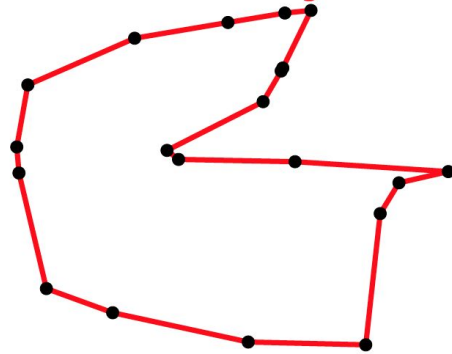
RL-PTR-NETs: 5.7

Ground Truth: tour length is 3.518



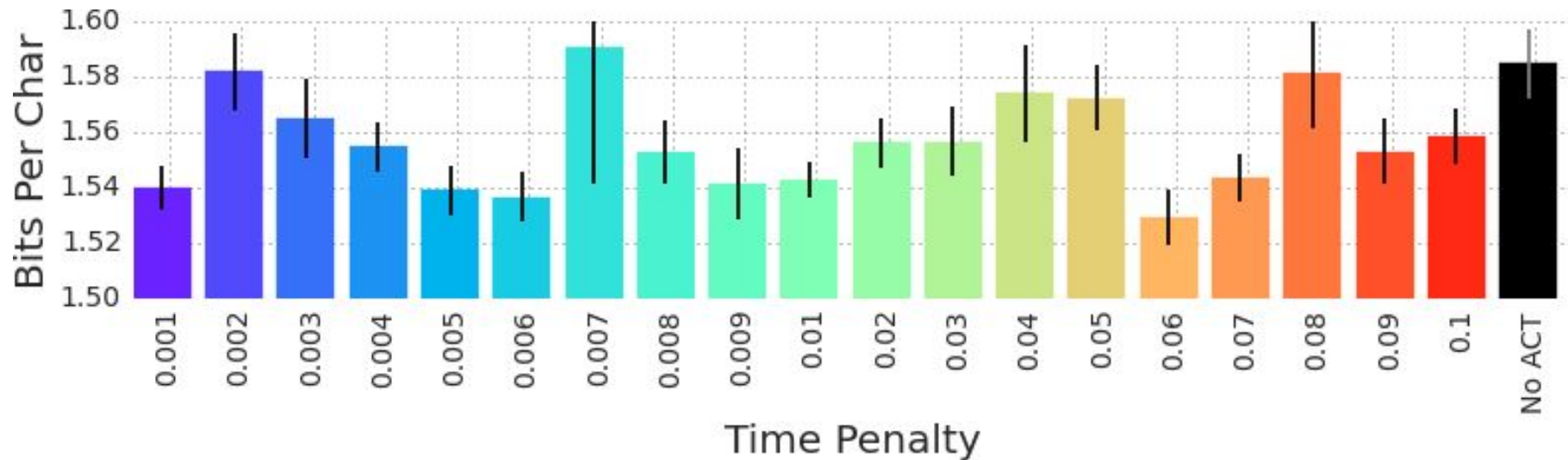
$n = 20$

Predictions: tour length is 3.523

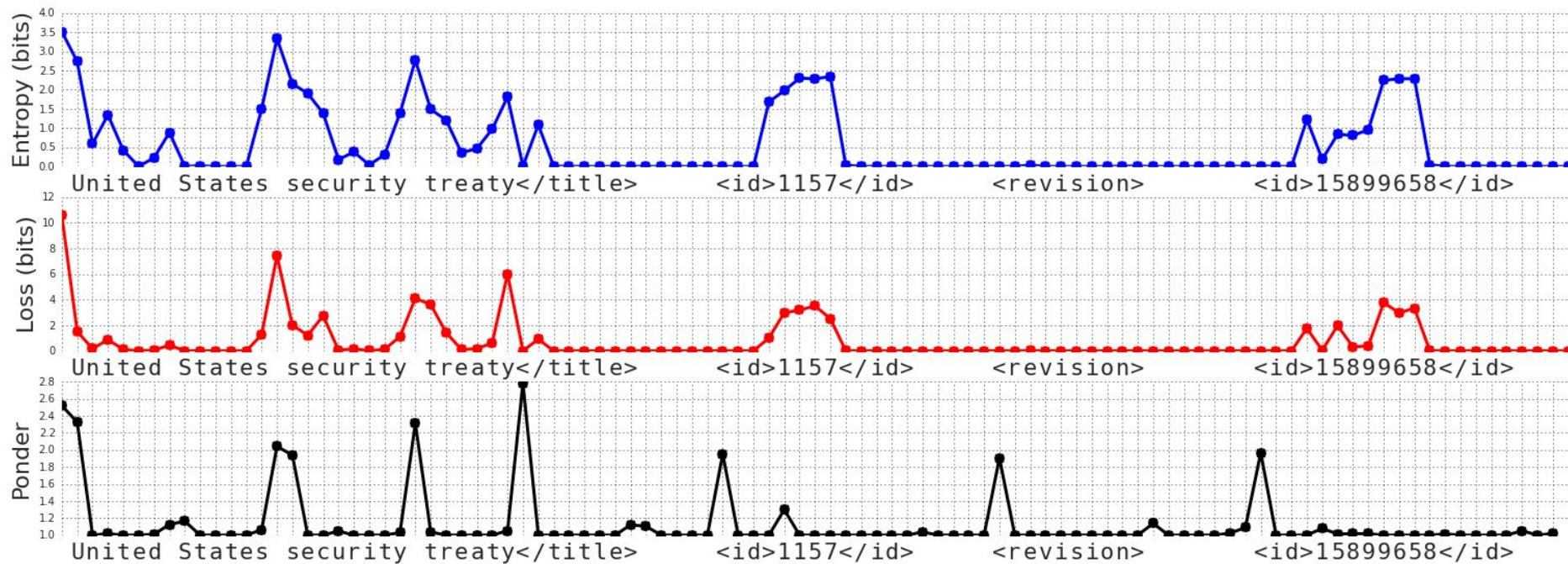


$n = 20$

# Wikipedia Character Prediction Results



# Pondering Wikipedia



# Word level LM

Baseline: 44 PPL

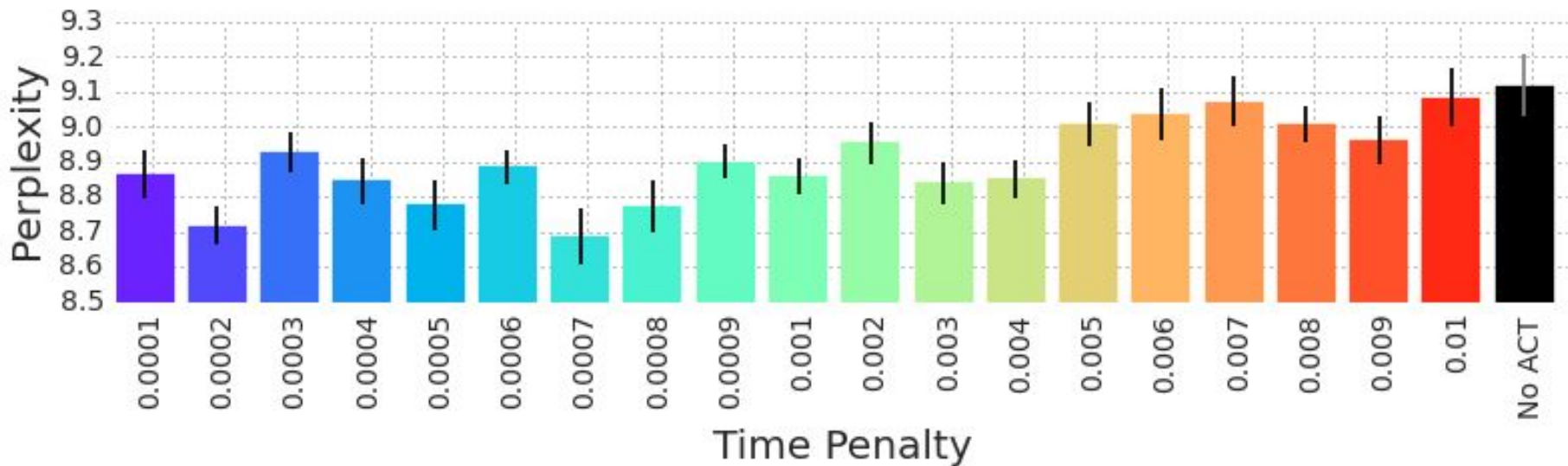
With fix pondering of 5: **39**

With pondering up to 10 (average **4.3**): **39**

<S>	2	
Elsewhere		3
,	4	
Nymex	4	
WTI	4	
crude	5	
<UNK>	6	
under	5	
the	4	
\$	3	
70	3	
a	4	
barrel	5	
mark	6	
,	5	
losing	6	
0.3	3	
per	2	
cent	4	
to	4	
\$	3	
69.57	4	
.	2	



# Character Level Machine Translation (BTEC)



# Machine Translation + ACT

Dataset: WMT14 test set, English to French

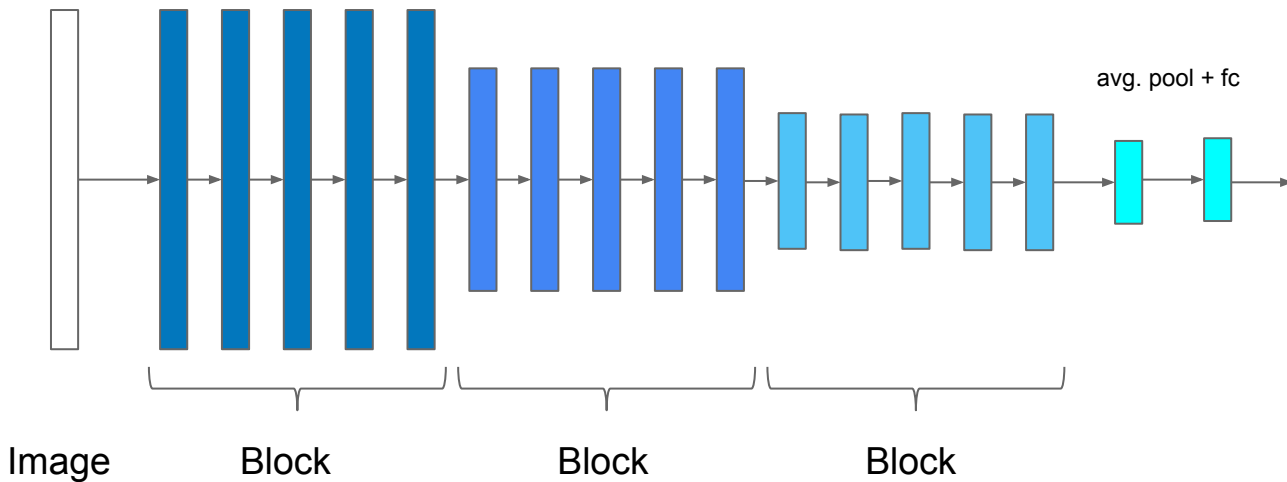
(SMT): 37.0 BLEU

Baseline AttLSTM: 3.4 PPL, 37.5 BLEU

AttLSTM + ACT (between input and output): 3.3 PPL, 37.6 BLEU

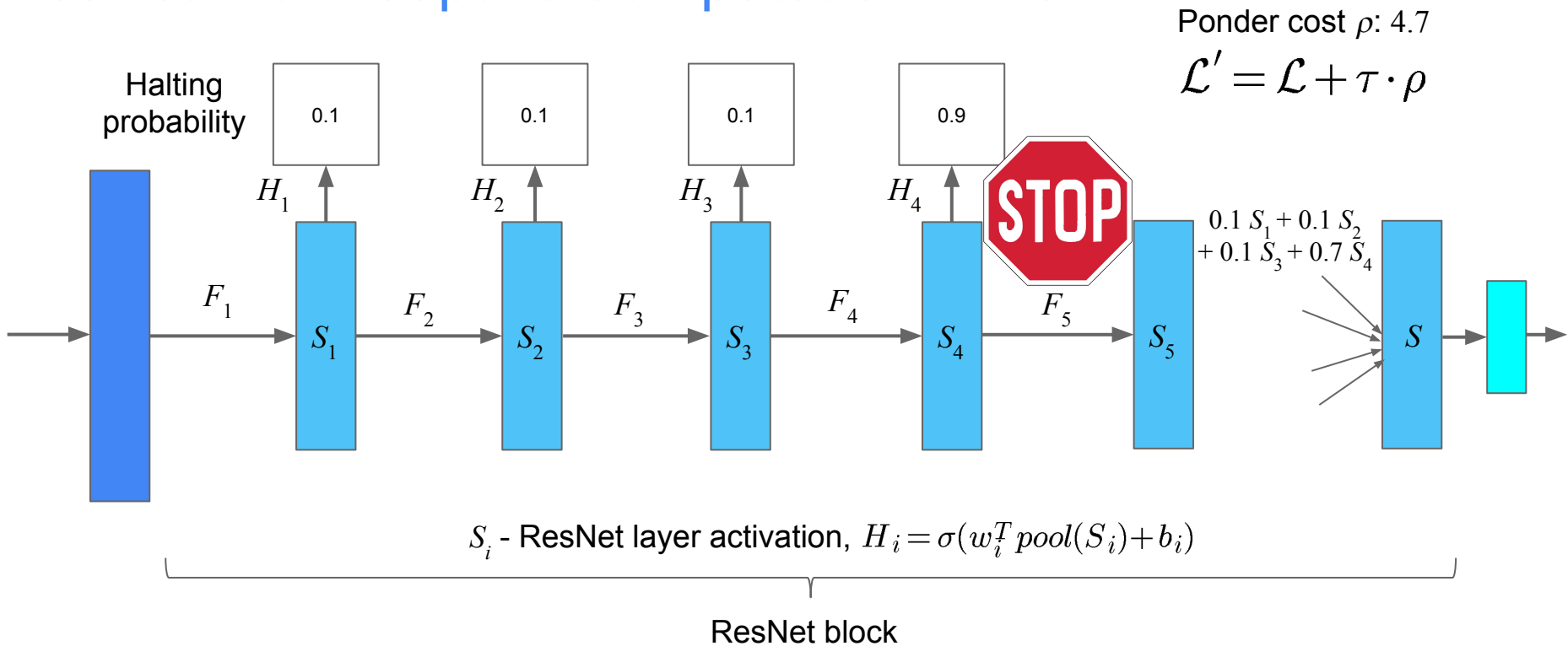
AttLSTM + ACT: **3.1** PPL, **38.3** BLEU

# Residual Network (ResNet)

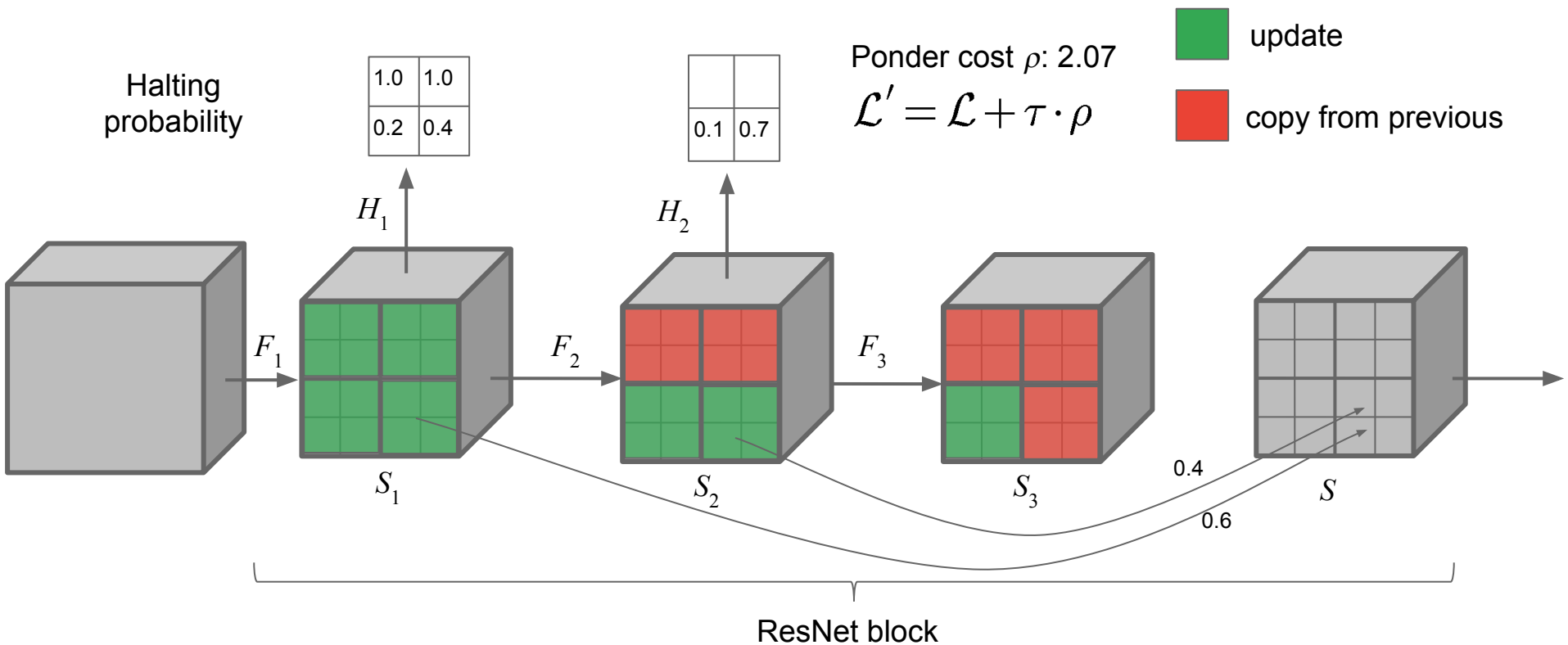


Residual layer:  $y = \mathcal{F}(x) + x$

# ResNet with Adaptive Computation Time

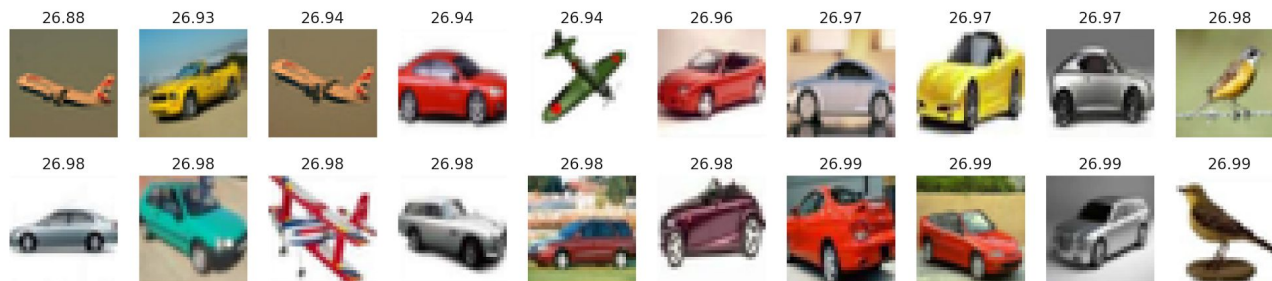


# ResNet with Spatially Adaptive Computation Time

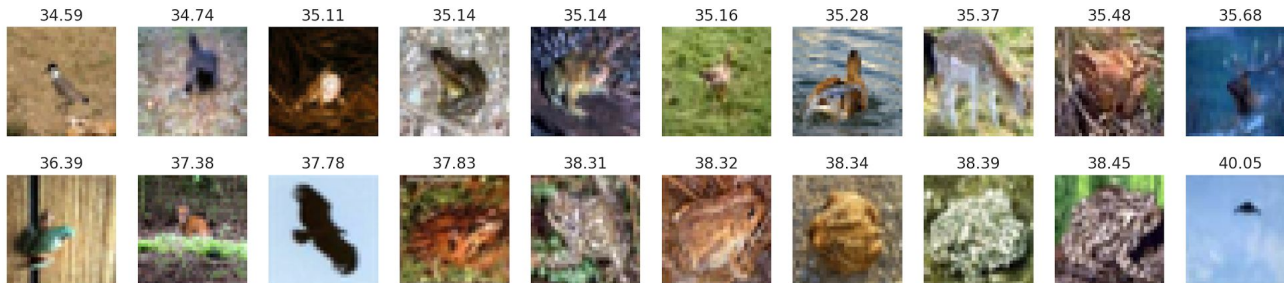


# CIFAR-10 ACT qualitative results

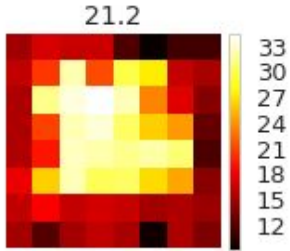
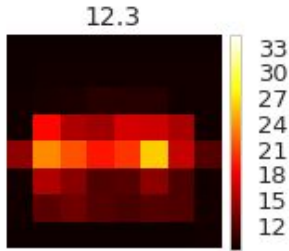
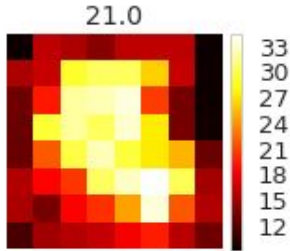
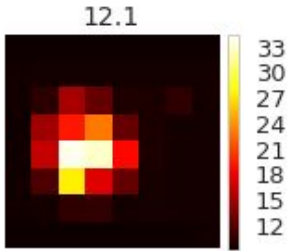
Low ponder cost



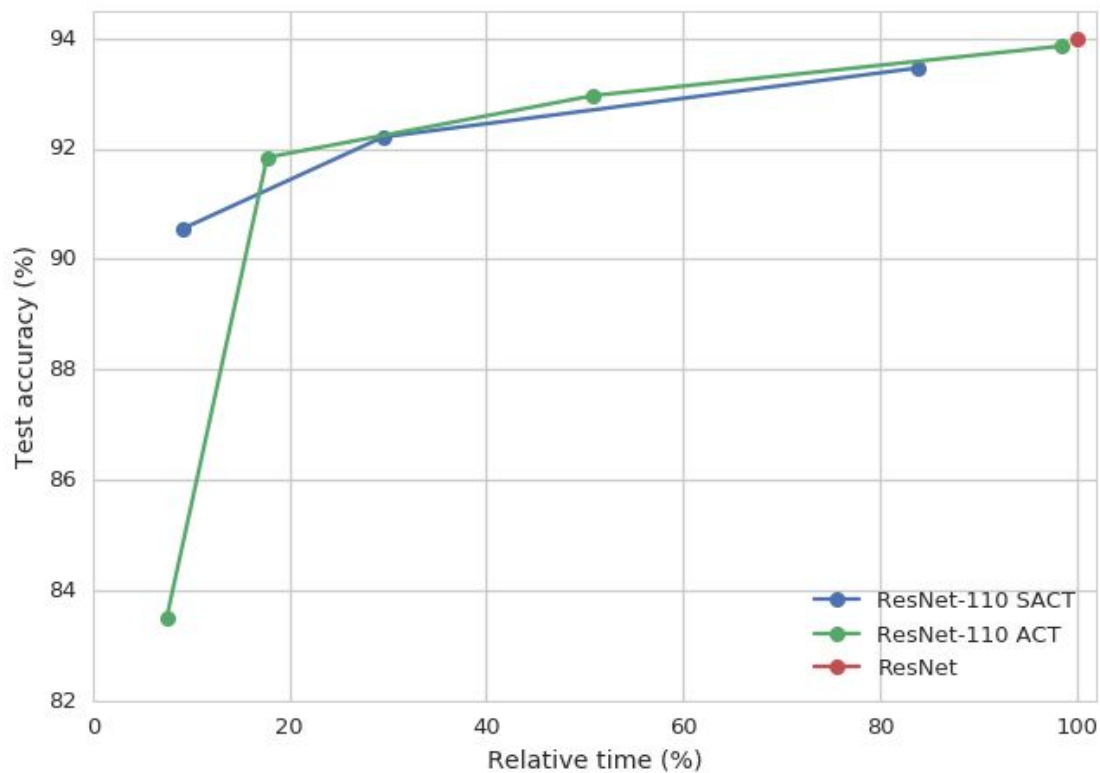
High ponder cost



# CIFAR-10 SACT qualitative results

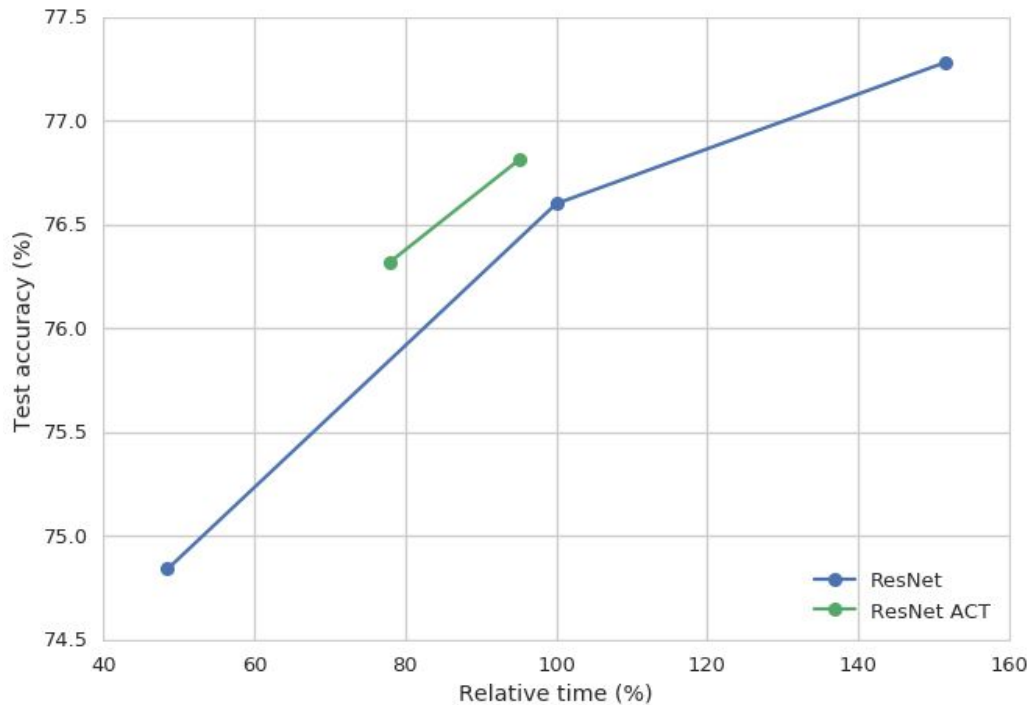


# CIFAR-10 ACT vs. SACT





# ImageNet ACT accuracy vs. time

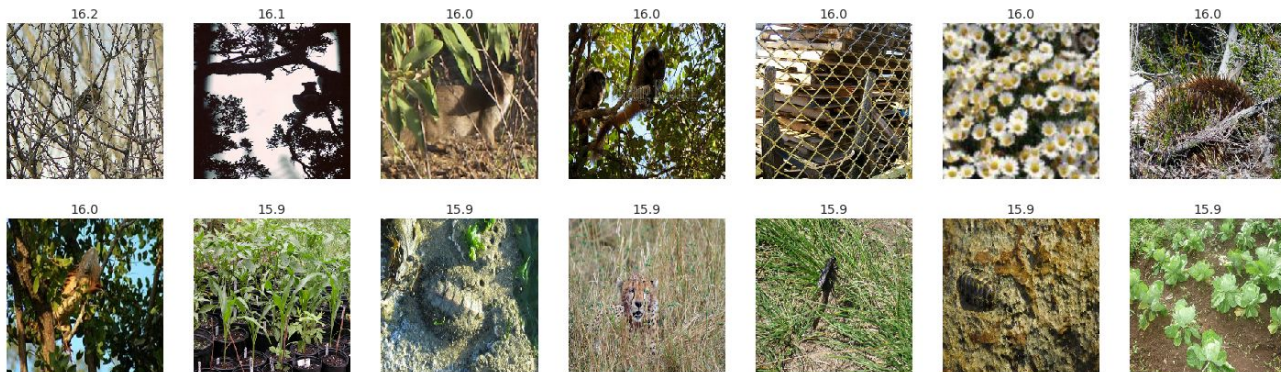


# ImageNet ACT qualitative results

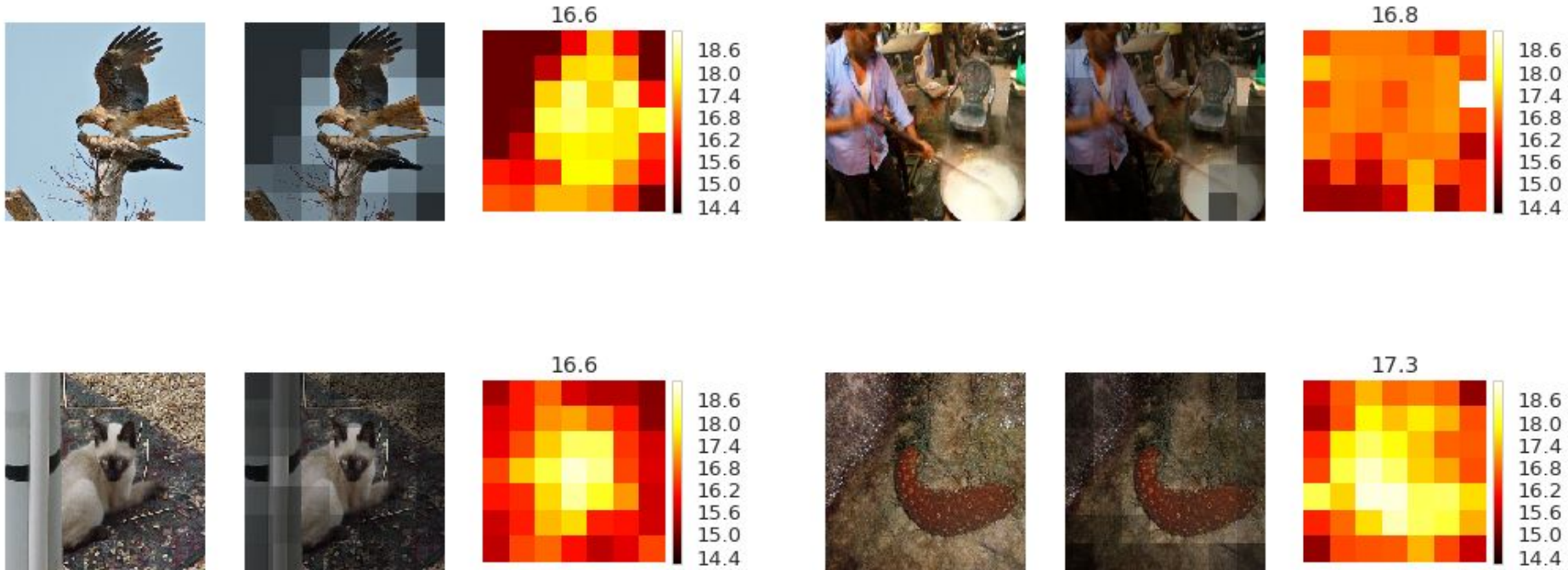
Low ponder cost



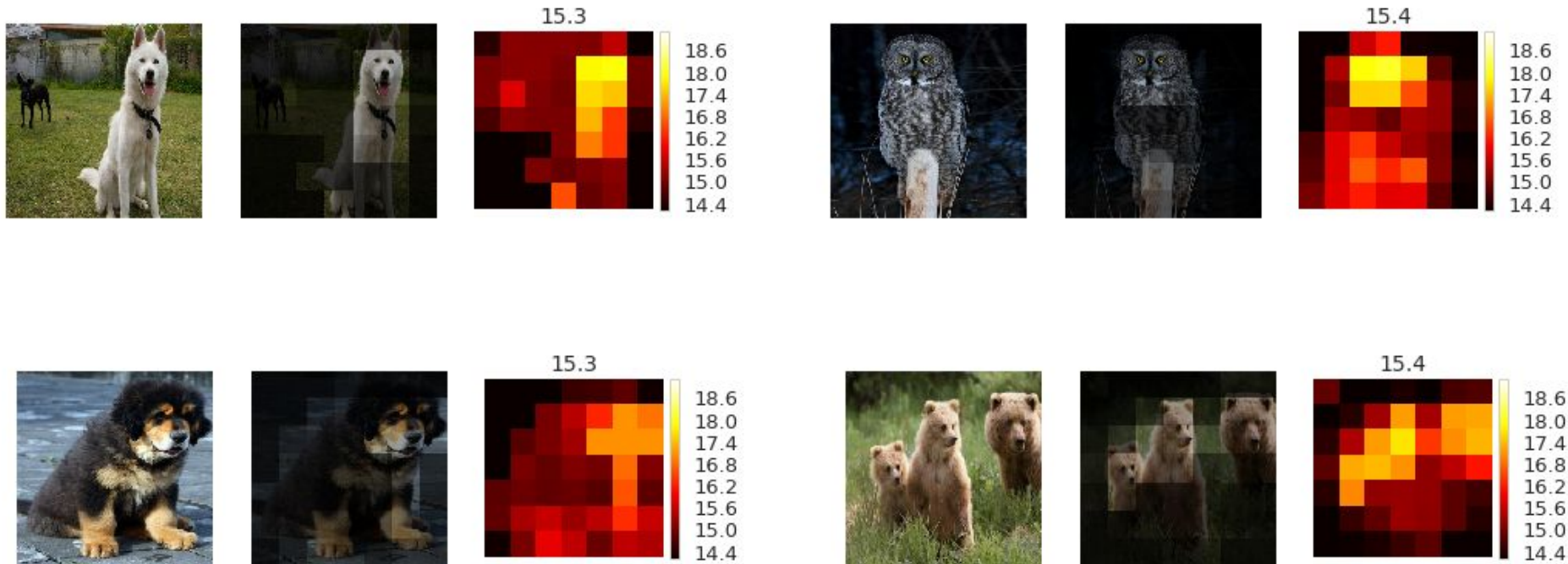
High ponder cost



# ImageNet SACT random examples



# ImageNet SACT low ponder cost examples



# ImageNet SACT high ponder cost examples

